SELF-SIMILAR PROBLEM OF THE NONISOTHERMAL BOUNDARY LAYER OF DILUTE SUSPENSIONS OF RIGID ELLIPSOIDAL PARTICLES

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A numerical solution is obtained for the self-similar problem of a plane nonisothermal boundary layer of dilute suspensions of rigid ellipsoidal particles.

Let us consider the plane steady laminar flow of a dilute equal-density suspension of rigid ellipsoidal particles near a hot or cold solid wall. Let us hence assume that the heat being liberated because of viscous dissipation is negligible (the Prandtl number of the suspension is Pr = 5.8); the coefficient of thermal conductivity of the suspension λ is a physical constant [1].

At high Reynolds numbers it is natural to consider the flow in the boundary-layer approximation. The equations of an isothermal boundary layer have been obtained in [2] and are

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\rho U\frac{dU}{dx}+\frac{\partial}{\partial y}\left[\left(\mu+\mu_1\frac{\langle n_x n_y\rangle}{\frac{\partial u}{\partial y}}+\mu_2\langle n_x^2 n_y^2\rangle+\mu_3\langle n_x^2+n_y^2\rangle\right)\frac{\partial u}{\partial y}\right], \tag{1}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0. \tag{2}$$

In the nonisothermal flow case, the system (1), (2) should be supplemented by the energy equation which is written in the approximation taken as follows:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=a\frac{\partial^2 T}{\partial y^2}. \tag{3}$$

Neglecting singularities of the interaction between the suspended particles and the walls, let us take the boundary conditions of the boundary-value problem (1)-(3) as

$$\begin{aligned} &u=v=0,\quad T=T_w(x)\quad \text{for}\quad y=0,\\ &u\to U(x),\quad T\to T_\infty\quad \text{for}\quad y\to\infty. \end{aligned} \tag{4}$$

Let us use the distribution functions of the elevations of the particle axis of symmetry, obtained for simple shear flow $u = Ky$, $v = w = 0$, $K = \text{const}$ [3, 4, 5] by replacing K by $\partial u/\partial y$ [2, 6], in taking the average in (1).

The nonisothermy of the flow is reflected in the temperature dependence of the dynamic coefficient of viscosity of the solvent μ_0 [7] and the coefficient of rotational particle diffusion D_r [8], which are in the rheological constants μ, μ_1, μ_2 and μ_3:

$$\mu_0=\mu_*\exp[-c(T-T_\infty)],$$

$$D_r=\frac{kT}{V\mu_0}\cdot\frac{p[(2p^2-1)\ln(p+\sqrt{p^2-1})-p\sqrt{p^2-1}]}{4(p^4-1)\sqrt{p^2-1}}. \tag{5}$$

As in the case of a Newtonian fluid, let us seek self-similar solutions of the problem (1)-(4) by giving the external flow velocity and the wall temperature distribution as

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$$U = cx^m, \quad T_w(x) = T_\infty + T_1 x^n \tag{6}$$

(c, m, T, and n are constants).

Introducing the stream function ψ and going over to the new variables ξ, Φ, θ, connected with c, x, y, ψ, T by the relationships

$$\xi = \sqrt{\frac{(m+1)c\rho}{2\mu_*}}\, y x^{\frac{m-1}{2}}, \quad \psi = \sqrt{\frac{2\mu_*}{(m+1)\rho}}\, c x^{m+1}\, \Phi(\xi),$$
$$T = T_\infty + T_1 x^n \theta(\xi), \tag{7}$$

it can be shown that (1)-(3) have a unique self-similar solution for m = 1/3, n = 0. The self-similar problem is formulated as follows:

$$\left[\frac{\mu_{\text{eff}}}{\mu_*} + \sqrt{\frac{2}{3}\cdot\frac{c^3\rho}{\mu_*}}\,\frac{\partial}{\partial K}\left(\frac{\mu_{\text{eff}}}{\mu_*}\right)\Phi''\right]\Phi''' + (T_w - T_\infty)\frac{\partial}{\partial T}\left(\frac{\mu_{\text{eff}}}{\mu_*}\right)\Phi''\theta' + \Phi\Phi'' = \frac{1}{2}[(\Phi')^2 - 1], \tag{8}$$

$$\theta'' + \Pr\Phi\theta' = 0; \tag{9}$$
$$\Phi = \Phi' = 0, \ \theta = 1 \ \text{for} \ \xi = 0,$$
$$\Phi' \to 1, \ \theta \to 0 \ \text{for} \ \xi \to \infty,$$

where

$$\mu_{\text{eff}} = \mu + \mu_1 \frac{\langle n_x n_y \rangle}{\frac{\partial u}{\partial y}} + \mu_2 \langle n_x^2 n_y^2 \rangle + \mu_3 \langle n_x^2 + n_y^2 \rangle. \tag{10}$$

The prime denotes the derivative with respect to the self-similar variable ξ.

The energy equation (9) has the following solution:

$$\theta = 1 - \frac{\int_0^\xi \exp\left(-\Pr\int_0^\xi \Phi(\bar\xi)\, d\bar\xi\right) d\xi}{\int_0^\infty \exp\left(-\Pr\int_0^\xi \Phi(\bar\xi)\, d\bar\xi\right) d\xi}. \tag{11}$$

The boundary-value problem for the integrodifferential equation obtained after substituting (11) into (8) was solved by numerical iteration. Taken as $\theta'(0)$ in each iteration is the value of this function computed by means of the distribution $\Phi(\xi)$ obtained in the previous iteration. Then the value of $\Phi''(0)$ was selected by a "ranging" method so that the boundary condition for Φ' would be satisfied as $\xi \to \infty$. The Cauchy problem for known $\theta'(0)$ and $\Phi''(0)$ was solved by a modified Runge–Kutta method with variable spacing [9].

The results of computing $\Phi'(\xi)$ and $\mu_{\text{eff}}(\xi)$ for aqueous suspensions of rigid ellipsoidal particles are represented in Fig. 1A, B for the following values of the parameters $r = \sqrt[3]{ab^2} = 10^{-6}$ m; a/b = 10, 25; φ = 0.01; Pr = 5.8; T_∞ = 300°K, solid lines – T_W = 280°K, dashed lines – T_W = 320°K.

The characteristics of isothermal and nonisothermal boundary layer of a solvent and suspension are compared in Table 1, where

$$A = \int_0^\infty [1 - \Phi'(\xi)]\, d\xi, \quad B = \int_0^\infty \Phi'(\xi)[1 - \Phi'(\xi)]\, d\xi, \quad C = \int_0^\infty \Phi'(\xi)\,\theta\, d\xi, \quad F = \left[\frac{\mu_{\text{eff}}}{\mu_*}\Phi''(\xi)\right]_{\xi=0}.$$

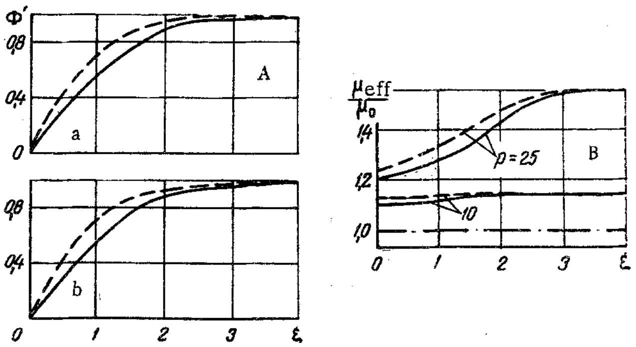

Fig. 1. Dependence of Φ' on ξ [a: p = 10, b: 25] (A) and dependence of μ_{eff}/μ_0 on ξ [dashed-dot line is for a Newtonian fluid] (B).

TABLE 1. Comparison between Isothermal and Nonisothermal Boundary-Layer Characteristics of a Solvent and Suspension for $Pr = 5.8$; $\mu_* = 8.595 \cdot 10^{-4}$ N · sec/m^2; $T_\infty = 300$°K.

	Suspension $p=10$					
	$T_w=320$ °K	Δ, %	$T_w=300$ °K	Δ, %	$T_w=280$ °K	Δ, %
$\Phi''(0)$	1,176	—4,60	$8,857 \cdot 10^{-1}$	—4,53	$5,941 \cdot 10^{-1}$	—4,45
$\theta'(0)$	—1,074	—1,41	—	—	$-9,268 \cdot 10^{-1}$	—1,44
ξ_{δ_d}	2,761	6,28	2,914	6,09	3,084	5,60
ξ_{δ_T}	1,543	1,32	—	—	1,720	1,36
$\mu_{eff}(0)$	$6,457 \cdot 10^{-4}$	11,23	$9,517 \cdot 10^{-4}$	10,71	$1,583 \cdot 10^{-3}$	10,17
$\mu_{eff}/\mu_*(0)$	$7,511 \cdot 10^{-1}$	11,23	1,107	10,71	1,841	10,17
A	$7,372 \cdot 10^{-1}$	5,87	$8,485 \cdot 10^{-1}$	5,49	$9,923 \cdot 10^{-1}$	4,97
B	$3,421 \cdot 10^{-1}$	6,25	$3,706 \cdot 10^{-1}$	5,85	$3,980 \cdot 10^{-1}$	5,31
C	$1,854 \cdot 10^{-1}$	—1,35	—	—	$1,598 \cdot 10^{-1}$	—1,44
F	$8,837 \cdot 10^{-1}$	6,12	$9,818 \cdot 10^{-1}$	5,83	1,094	5,27
	Suspension $p=25$					
	$T_w=320$ °K	Δ, %	$T_w=300$ °K	Δ %	$T_w=280$ °K	Δ, %
$\Phi''(0)$	1,112	—9,74	$8,522 \cdot 10^{-1}$	—8,14	$5,726 \cdot 10^{-1}$	—7,91
$\theta'(0)$	—1,062	—2,54	—	—	$-9,157 \cdot 10^{-1}$	—2,62
ξ_{δ_d}	3,087	18,84	3,233	17,68	3,408	16,68
ξ_{δ_T}	1,561	2,53	—	—	1,741	2,56
$\mu_{eff}(0)$	$7,255 \cdot 10^{-4}$	24,99	$1,060 \cdot 10^{-3}$	23,33	$1,749 \cdot 10^{-3}$	21,74
$\mu_{eff}/\mu_*(0)$	$8,441 \cdot 10^{-1}$	24,99	1,233	23,33	2,035	21,74
A	$7,885 \cdot 10^{-1}$	13,24	$9,008 \cdot 10^{-1}$	11,99	1,047	10,78
B	$3,720 \cdot 10^{-1}$	15,54	$3,997 \cdot 10^{-1}$	14,16	$4,273 \cdot 10^{-1}$	13,05
C	$1,829 \cdot 10^{-1}$	—2,65	—	—	$1,578 \cdot 10^{-1}$	—2,62
F	$9,394 \cdot 10^{-1}$	12,82	1,051	13,30	1,165	12,11

The relative deviations of the boundary-layer parameters of the suspension from the corresponding Newtonian fluid (solvent) parameters are denoted by Δ, %.

The results presented show that the boundary-layer thicknesses δ_d, δ_T, δ^*, δ^{**}, δ and the friction stress τ_w in the suspension are greater than in the solvent, where this difference increases with the growth in the elongation $p = a/b$ of the suspended particle.

NOTATION

U, velocity on outer limit of boundary layer; $Pr = \mu_*/\rho a$, Prandtl number of the suspension; ρ, density of the suspension; u, v, velocity components of the suspension in an orthogonal x0y coordinate system ordinarily used in boundary-layer theory; μ, μ_1, μ_2, μ_3, rheological constants, known functions [2] of parameters characterizing the suspension; n_x, n_y, components of the unit orientation vector directed along the axis of symmetry of the suspended particle; ⟨ ⟩, symbol of the averaging performed by using the distribution function of the elevations of the axis of rotation of the suspended particle; a, coefficient of thermal diffusivity of the suspension; T, absolute temperature of the suspension; T_w, temperature of the solid boundary; T_∞, temperature of the suspension in the outer flow; k, Boltzmann constant; V, volume of the ellipsoidal particle; $p = a/b$; a, b, the major and minor semiaxes of the ellipsoidal particle; μ_*, coefficient of dynamic viscosity of the solvent for $T = T_\infty$; ξ, dimensionless independent variable; Φ, dimensionless stream function; θ, dimensionless temperature; μ_{eff}, effective viscosity of the suspension; δ_d, dynamic boundary-layer thickness; δ_T, thickness of a temperature boundary-layer; δ^*, displacement thickness; δ^{**}, loss of momentum thickness; δ_T^{**}, "mixed thickness"; τ_w, friction stress on the wall.

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VISCOELASTIC BEHAVIOR OF MINERAL OILS AT HIGH PRESSURE

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Results of an experimental study of shear viscoelasticity of mineral oils and a method for approximating liquid relaxation spectra by a generalized Maxwell model are presented.

The stressed state of an oil layer in a heavily loaded rapidly moving elastohydrodynamic friction-pair contact can be described by the assumption of delay in establishment of equilibrium viscosity upon a sharp change in pressure in the contact zone [1, 2, 3].

The representation of liquid structural equilibrium-delay processes by the Frenkel'–Obraztsov model for gradual pressure change [2] leads to the expressions for viscosity at the contact:

$$\eta = \eta_0 \exp \alpha P_1 \exp(-\beta\alpha P_1) \tag{1}$$

and delay time:

$$t_{\text{ret}} = \frac{\eta_0}{G_\infty} \exp \alpha P_1. \tag{2}$$

The parameter β is defined in [2] from the equation

$$\frac{G_\infty t}{\eta_0 \exp \alpha P_1} = \text{Ei}(\beta\alpha P_1) - \text{Ei}(\beta_1 \alpha P_1). \tag{3}$$

Here $\text{Ei}(\beta_1 \alpha P_1)$ is an exponential integral, which considers the loading prehistory.

In Eqs. (2) and (3) there appears the value of the instantaneous shear modulus of elasticity G_∞, which can be obtained from study of liquid relaxation spectra.

It is known [4] that mineral oils have continuous relaxation spectra encompassing not less than 5–8 frequency decade. A phenomenological representation of this behavior in mineral oils is possible within the framework of the generalized Maxwell model. For this model the most general form of the complex modulus of elasticity expanded into the relaxation time spectrum was employed:

$$G = G_\omega + jG_N = \sum_{i=1}^{n} \frac{\omega^2 \tau_i^2}{1 + \omega^2 \tau_i^2} G_i + j\omega \sum_{i=1}^{n} \frac{\eta_i}{1 + \omega^2 \tau_i^2}. \tag{4}$$

The components of the complex modulus of elasticity G_ω and G_N were measured at pressures to $6 \cdot 10^8$ N/m^2 at temperatures of 17-100°C by quartz torsional oscillation resonators at frequencies of 23, 43, 80, and 126 kHz by the method described in [5]. Since this frequency interval does not include the range

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